Vatsalya Sr.Sec.School Vidisha
Class X
Mathematics

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES 3

In this chapter, you have studied the following points:

1. Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1 + b_1 \neq 0$, $a_2 + b_2 \neq 0$.

2. A pair of linear equations in two variables can be represented, and solved, by the:

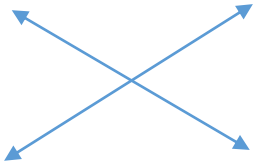
(i) graphical method

(ii) algebraic method

3. Graphical Method :

The graph of a pair of linear equations in two variables is represented by two lines.

(i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



(ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.

(iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.

4. Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :

(i) Substitution Method :-

To understand the substitution method more clearly, let us consider it stepwise:

Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

Step 2 : Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved. Sometimes, as in Examples 9 and 10 below, you can get statements with no variable. If this statement is true, you can conclude that the pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.

Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

(ii) Elimination Method :-

Let us now note down these steps in the elimination method :

Step 1 : First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2 : Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3. If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions. If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.
Step 3 : Solve the equation in one variable (x or y) so obtained to get its value.
Step 4 : Substitute this value of x (or y) in either of the original equations to get the value of the other variable

(iii) Cross-multiplication Method :-
For solving a pair of linear equations by this method, we will follow the following steps :
Step 1 : Write the given equations in the form (1) and (2).
Step 2 : Taking the help of the following formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Step 3 : Find x and y, provided $a_1b_2 - a_2b_1 \neq 0$

5. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_1}$: In this case, the pair of linear equations is consistent.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case, the pair of linear equations is inconsistent.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case, the pair of linear equations is dependent and consistent.

6. There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

Some extra questions for solving :

1. $\frac{5}{x+y} + \frac{2}{x-y} = 2$

   $\frac{9}{x+y} - \frac{4}{x-y} = 1$

2. $\frac{1}{2(2x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$

   $\frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$

3. $\frac{44}{x+y} + \frac{30}{x-y} = 10$

   $\frac{55}{x+y} + \frac{40}{x-y} = 13$

4. $\frac{4}{x} + 5y = 7$

   $\frac{3}{x} + 4y = 5$

5. $\frac{2}{x} + \frac{3}{y} = 13$

   $\frac{5}{x} - \frac{4}{y} = -2$

6. $\frac{5}{x-1} + \frac{1}{y-2} = 2$

   $\frac{6}{x-1} - \frac{3}{y-2} = 1$

7. $\frac{x}{a} + \frac{y}{b} = 2$

   $ax - by = a^2 - b^2$

8. $\frac{x}{a} + \frac{y}{b} = a + b$

   $\frac{x}{a^2} + \frac{y}{b^2} = 2$

9. $\frac{x}{a} = \frac{y}{b}$

10. $\frac{5}{x+y} - \frac{2}{x-y} = -1$

$ax + by = a^2 + b^2$ $\qquad\qquad$ $\frac{5}{x+y} - \frac{7}{x-y} = 10$

11. A and B each have a certain number of mangoes. A says to B, " if you give 30 of your mangoes, I will have twice as many as left with you. "B replies, "if you give me 10, I will have thrice as many as left with you. "How many mangoes does each have?

12. On selling a T.V.at 5% gain and fridge at 10% gain, a shopkeeper gains Rs 2000. But if he sells the T.V. at 10% gain and the fried at 5% loss. He gains Rs 1500 on the transaction. Find the actual prices of T.V. and fridge.

13 A two digit number is obtained by either multiplying sun of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2 find the number.

14 If three times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also, if seven times the smaller number is divide by the larger one we get 5 as quotient and 1 as remainder. Find the numbers.

15 Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

16 Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three the age of the son. Find the present ages of father and son.

17 X takes 3 hours more than Y to walk 30 km. but if X doubles his pace, he pace he is ahead of Y by $1\frac{1}{2}$ hours. Find their speed of walking.

18 After covering a distance of 30 km with a uniform speed there is some defect in a train engine and therefore, its speed is reduced to 4/5 of its original speed. Consequently, the train reaches its destination late by 45 minutes. Had it happened after covering 18 kilometers more, the train would have reached 9 minutes earlier. Find the speed of the train and the distance of journey.

19 On selling a tea – set at 5% loss and a lemon – set at 15% gain, a crockery seller gains Rs 7 if he sells the tea – set at 5% gain and the lemon – set at 10% gain, he gains, Rs 13. Find the actual price of the tea set and the lemon – set.

20 The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs 1250, find their incomes.

QUADRATIC EQUATIONS 4

In this chapter, you have studied the following points:

1. A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

3. If we can factorise $ax^2 + bx + c$, $a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

4. A quadratic equation can also be solved by the method of completing the square.

5. Quadratic formula: The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

Thus, if $b^2 - 4ac \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula for finding the roots of a quadratic equation is known as the **quadratic formula**.

6. A quadratic equation $ax^2 + bx + c = 0$ has

(i) two distinct real roots, if $b^2 - 4ac > 0$,

(ii) two equal roots (i.e., coincident roots), if $b^2 - 4ac = 0$, and

(iv) no real roots, if $b^2 - 4ac < 0$.

Some important questons:

1. $x^2 - 4\sqrt{2}x + 6 = 0$

2. $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

3. $\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$

4. $\frac{x+3}{x+2} - \frac{1-x}{x}\frac{17}{4}$

5. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}$, $x \neq 3, x \neq -3$

6. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$, $x \neq 0$

7. $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}$, $x \neq 1, -1$

8. $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}$, $x \neq -\frac{1}{2}, 1$

9. $3x^2 - 14x - 5 = 0$

10. $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

11. One – fourth of a herd of camels was seen in the forest. Twice the square root of the hard had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

12. O Girl! Out of a group of swans, $\frac{7}{2}$ times the square root of the number are playing on the shore of a tank. The two remaining ones are playing, with amorous fight, in the water. What is the total number of swans?

13. A two – digit number is such that the product of is 8. When 18 is subtracted from the number ,the digit interchange their place .find the number.

14. Two numbers differ by 3 and their product is 504 find the numbers.

15. Two trains leave a railway satiation at the same time. The first train travels due west and the second train due month. The first train travels 5 km/hr. faster than the second train. If after two hours, they are 50 km. apart, find the average speed of train.

16. Swati can row boat at speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5.25km upstream than to return downstream, find the speed of the stream.

17. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present age.

18. There is a square field whose sides is 44 m. A square flower bad is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bad and gravelling the path at Rs 2.75 and Rs 1.50 per square metre, respectively, is Rs 4904. Find width of the gravel path.

19. A swimming pool is filled with pipes with uniform flow. The first two pipes operating simultaneously fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

20. The angry Arjun some arrows for fighting with Bheeshm. With half the arrows, he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the rath driver of Bheeshm. With one arrow each he knocked down respectively the rath, flag and the bow of Bheeshm. Finally, with one more than four times the square root of arrows he laid Bheeshm unconscious on an arrow bed. Find the total number of arrows Arjun had.